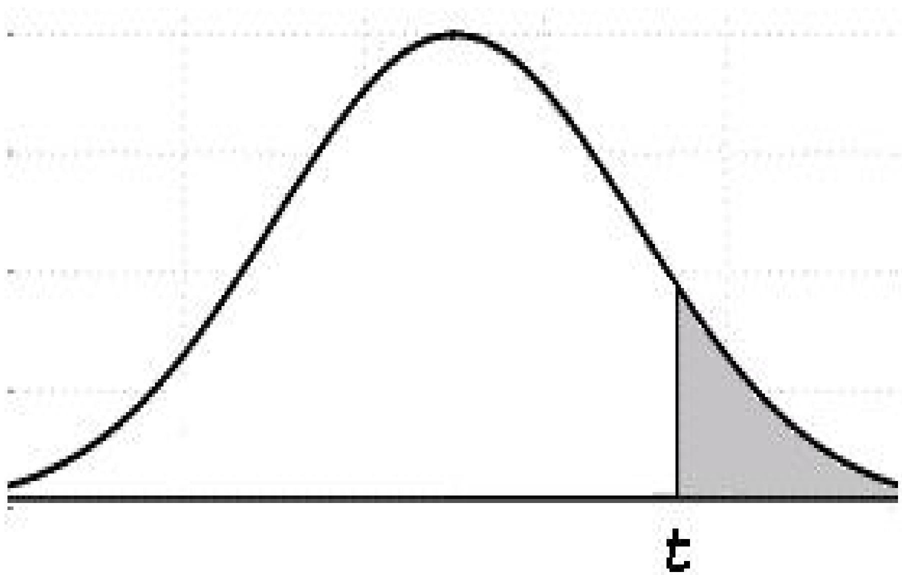
t – Test Statistic

Test of Difference

We use t when we are trying to determine if there is a Statistically Significant difference between two Means

T distribution of probabilities



You must use the t-distribution table when working problems when the population standard deviation (σ) is not known and the sample size is small ($n<30$)

t - Test

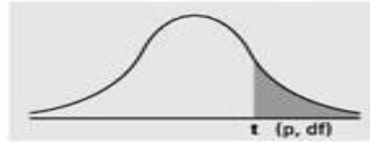
t is a Test Statistic used in tests involving the **difference between two Means**

(t is also known as “Student’s t” or the “t-statistic)

t - Test

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{N}}}$$

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

t - Test

- t is a measure of how likely it is that a **difference in Means** is Statistically Significant

- As with all Test Statistics, we compare t to its Critical Value. The value of t is calculated from Sample data

- The value of t-critical is determined by the value selected for Alpha, the Significance Level, and the appropriate t-Distribution

t - Test

- A large value for t makes it more likely to be larger than t-critical, and so makes it more likely that there is a Statistically Significant difference in the Means

t - Test

- Since the **<u>difference between the Means</u>** and the **<u>Sample Size, n</u>**, are *in the numerator*, larger values for either of these would make t larger.
- Since it's actually the square root of n that is in the numerator, an increase in the difference between the Means would have much more of an effect than a proportional increase in the Sample Size

t - Test

- Since the **Standard Deviation** is *in the denominator*, a larger Variation in the Sample(s) will make **(t)** smaller

t - Test

- For any value of the Test Statistic, we can determine the Probability of that value same as we did with the **z-test**
- **….. the difference between two Means**
  - **One Mean is always the Mean of a Sample**
  - **The Second Mean can be either**
    - **A specified Mean, such as a target Mean, a historical Mean or an estimate, or**
    - **The Mean of a Sample from a different Population or Process than the first Mean, or**
    - **A second Mean from the same test subjects (e.g., before and after some event)**

These three different types of the second Mean correspond to three different t-tests

t-test	Mean 1	Mean 2
1-Sample	Sample from a Population or Process	Specified Mean (a target, an estimate, or a historical value)
2-Sample	Sample from a Population or Process	Sample from a different Population or Process
Paired	one half of a Sample of paired data, e.g., score before training	the other half of the Sample of paired data for the same test subjects, e.g., score after training

t - Test

- **Degree of Freedom**
  - For a **single** sample t-test, **df** = n – 1, where n is the Sample Size
  - In the **2-Sample** t-test, **df** = n1 + n2 − 2

t - Test

Assumptions for the data are:

- For one sample = Normality, and
- For the 2-Sample t-test – equal Variance

1-Sample t-test

- The 1-Sample t-test compares a calculated Sample Mean to a Mean we specify
- We need to compare two Means; where does the other Mean come from? **It can come from anywhere**

1-Sample t-test

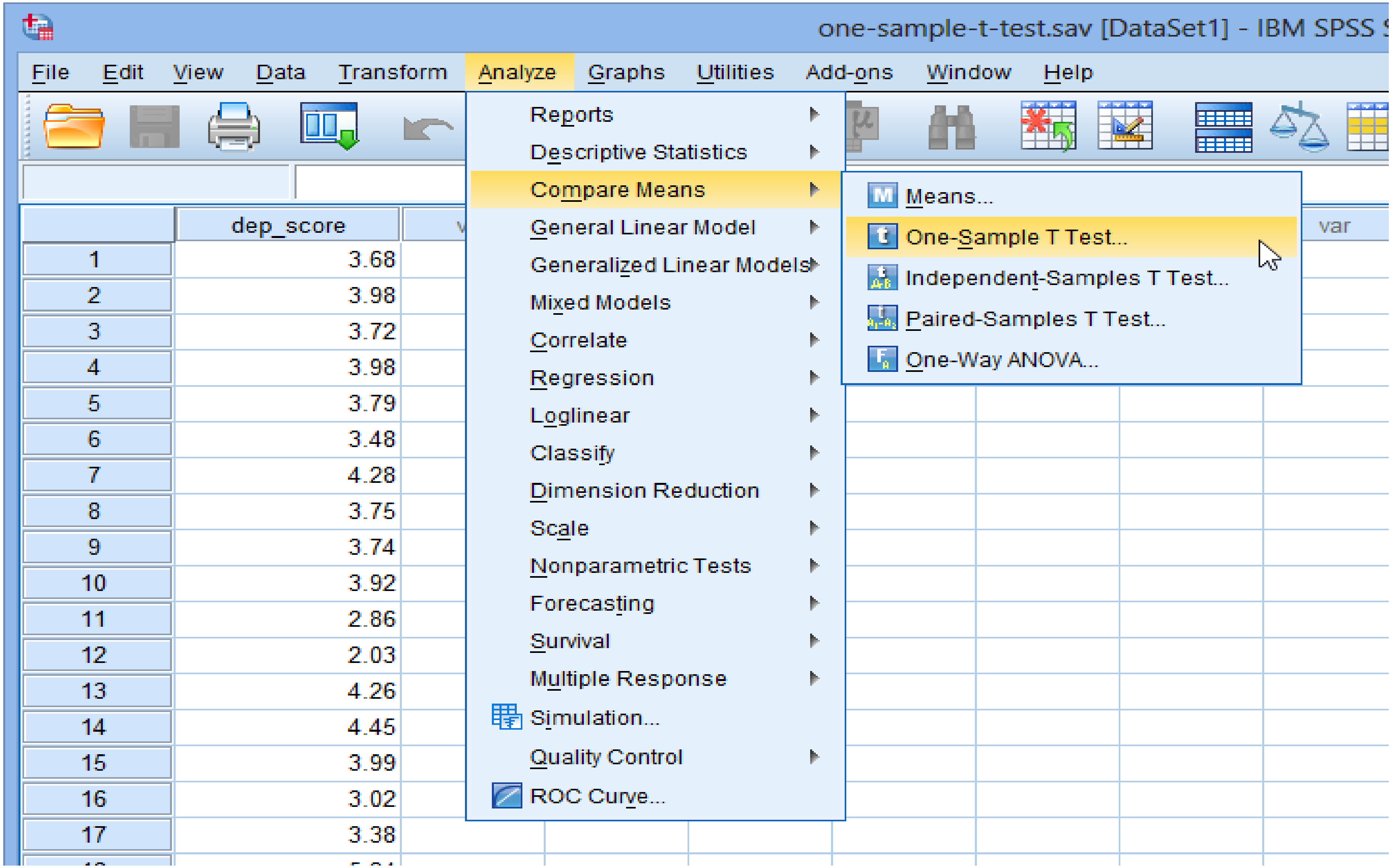
- A Mean calculated outside our test. For example, we compare Mean exam scores for our high school class with the national average
- An estimated or hypothesized Mean
- A historical Mean. For example, we may suspect that the Mean of our Process has drifted slightly from what it's always been
- A target Mean

1-Sample t-test

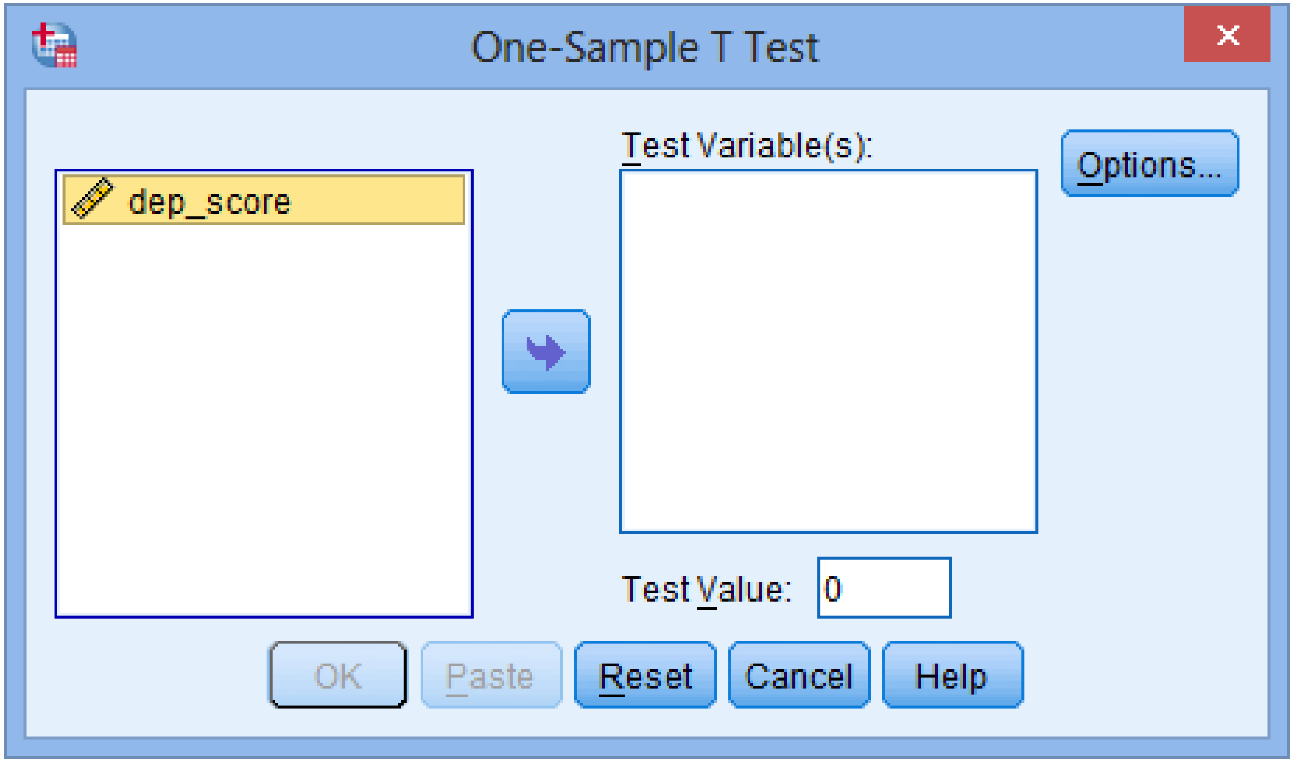
- **Hypothesis:** I assume that the mean biostatistics grade is not the same as the mean biostatistics grade among UAE universities

- **Question:**
  - Does the mean grade differ from mean grade among UAE universities? **(two-tailed)**
  - Does the mean grade differ greater than the mean UAE universities grade? **(one tailed)**

1-Sample t-test



1-Sample t-test



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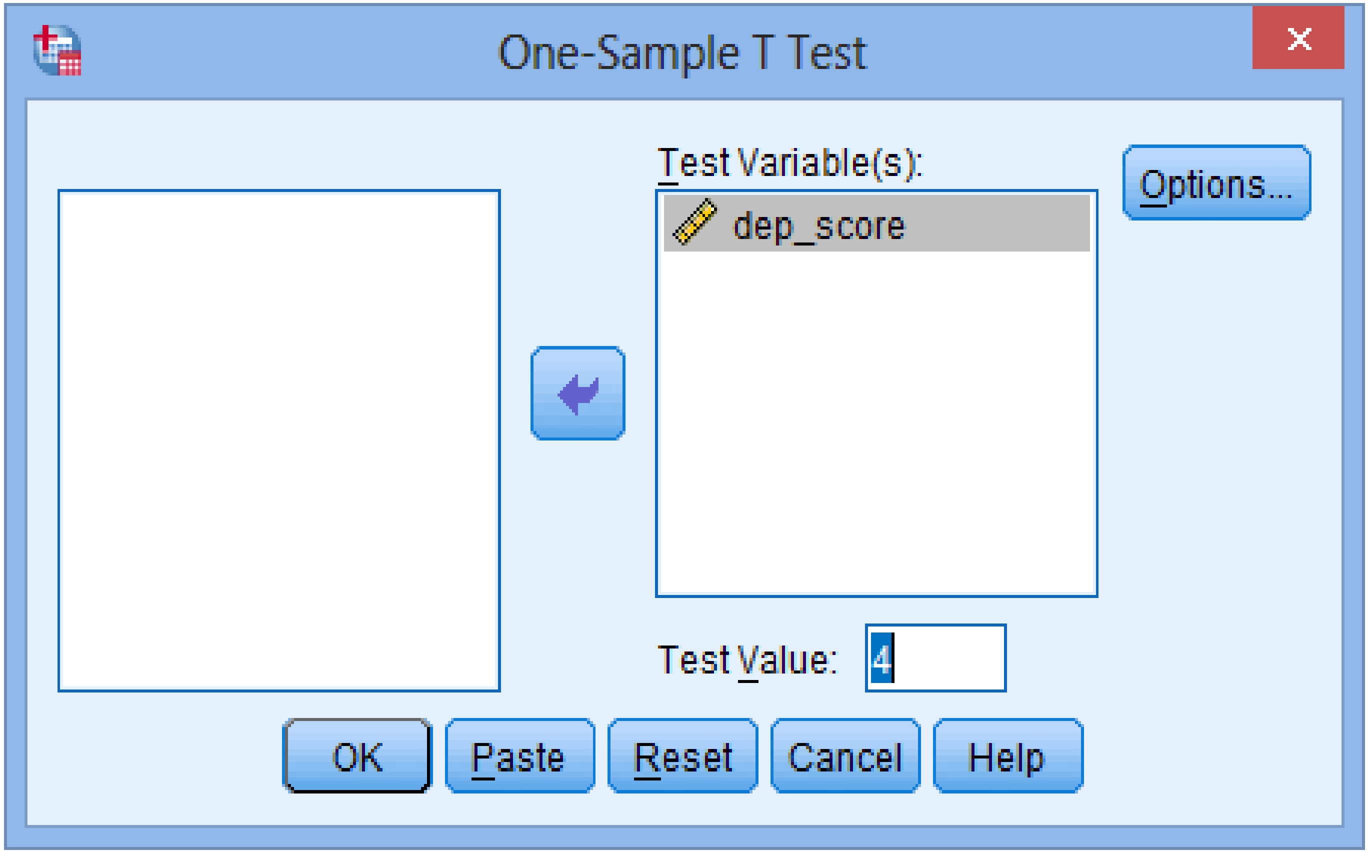
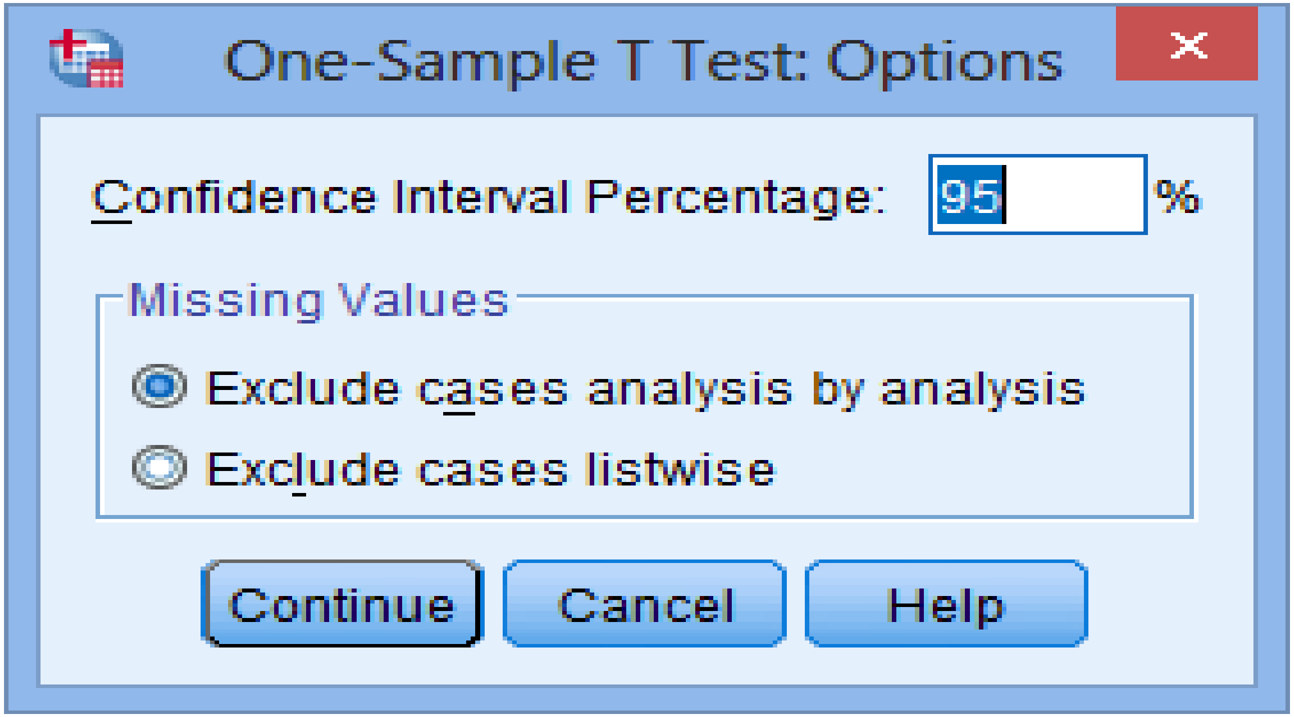


1-Sample t-test



1-Sample t-test

By default, SPSS Statistics uses 95% confidence intervals (labelled as the Confidence Interval Percentage in SPSS Statistics). This equates to declaring statistical significance at the $p < .05$ level. If you wish to change this you can enter any value from 1 to 99.

For example, entering "99" into this box would result in a 99% confidence interval and equate to declaring statistical significance at the $p < .01$ level

1-Sample t-test

One-Sample Test

	Test Value = 4					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
dep_score	-2.381	39	.022	-.27750	-.5132	-.0418

1-Sample t-test

One-Sample Test

	Test Value = 66.5 (A)					
	(B) t	(C) df	(D) Sig. (2-tailed)	(E) Mean Difference	95% Confidence Interval of the Difference (F)	
					Lower	Upper
Height	5.810	407	.000	1.53176	1.0135	2.0501

1-Sample t-test

- (A) **Test Value**: The number we entered as the test value in the One-Sample T Test window.

- (B) **t Statistic**: The test statistic of the one-sample *t* test, denoted *t*. In this example, $t = 5.810$. Note that *t* is calculated by dividing the mean difference (E) by the standard error mean (from the One-Sample Statistics box).

1-Sample t-test

- (C) **df**: The degrees of freedom for the test. For a one-sample t test, df = n - 1; so here, df = 408 - 1 = 407.

- (D) **Sig. (2-tailed)**: The two-tailed p-value corresponding to the test statistic.

1-Sample t-test

- (E) **Mean Difference**: The difference between the "observed" sample mean (from the One Sample Statistics box) and the "expected" mean (the specified test value (A)). The sign of the mean difference corresponds to the sign of the *t* value (B). The positive *t* value in this example indicates that the mean height of the sample is greater than the hypothesized value (66.5).

- (F) **Confidence Interval for the Difference**: The confidence interval for the difference between the specified test value and the sample mean.