

# Chi-Square

$$\chi^2$$

# Chi-Square

Is a Test Statistic which is very versatile in the types of data it can handle: Discrete, Continuous, nonNormal, Categorical

Like  $z$ ,  $t$ , and  $F$ , *the Chi-square is a* Test Statistic which has a Distribution or Distributions with known Probabilities for every value of the Test Statistic

# Chi-Square

Chi-Square Test	df
for Goodness of Fit	$n - 1$
for Independence	$(r - 1)(c - 1)$
for Variance	$n - 1$

# Chi-Square

## Chi-Square Test for the Variance

- This test compares the Sample Variance to a value of Variance which we specify
- The test tells us whether there is a Statistically Significant difference between the Sample Variance and the specified Variance. It is analogous to the 1-Sample t-test for Means.

# Chi-Square

$$\chi^2 = (n - 1) \frac{\sigma^2}{s^2}$$

# Chi-Square

- **Chi-Square Test for Goodness of Fit**
  - The Chi-Square ( $\chi^2$ ) Test for Goodness of Fit is a 1-way test of a Categorical Variable
  - The Test determines whether there is a Good Fit between Expected (E) Frequencies which we have specified or a Distribution and Observed (O) Counts from data

# Chi-Square

- Null Hypothesis ( $H_0$ ): There is no Statistically Significant difference between the Observed Counts and the Expected Frequencies. Therefore, there is a Good Fit.

# Chi-Square

The Chi-Square Test for Goodness of Fit has certain minimum size requirements (test Assumptions)

- Every Expected Frequency must be 1 or greater
- and no more than 20% of the Expected Frequencies can be below 5
- *If either of these Assumptions are not met, increasing the Sample Size will often help*

# Chi-Square

**Test Statistic:**  $\chi^2 = \sum \frac{(O - E)^2}{E}$

**Critical Value:** determined by  $\alpha$ , and the Degrees of Freedom, df.

**df =  $n - 1$** , where  $n$  is the number of categories.

# Chi-Square

## Results of the Test

- If  $\chi^2 \geq \chi^{2\text{-critical}}$ , or equivalently,  $p \leq \alpha$ ,
  - There is a Statistically Significant difference
  - There is not a Good Fit.
  - Reject  $H_0$ , the Null Hypothesis.

# Chi-Square

- If  $\chi^2 < \chi^2\text{-critical}$ , or equivalently,  $p > \alpha$ ,
  - There is not a Statistically Significant difference.
  - There is a Good Fit.
  - Fail to Reject (Accept)  $H_0$ , the Null Hypothesis

# Chi-Square

- **CHI-SQUARE TEST FOR INDEPENDENCE**
  - The Test Statistic, Chi-Square ( $\chi^2$ ), can be used to test whether two Categorical Variables are Independent or Associated

# Chi-Square

- Independence is determined between Variables not between values (category names) of the Variables
- For example, “Is Ice Cream flavor preference dependent on Gender?”

# Chi-Square

- If two Categorical Variables are Independent, then the Observed Frequencies (Counts) of the different values of the Variables should be Proportional
- **Important: the Expected Frequencies in each cell must be 5 or larger**

# Chi-Square

**Juice Study: Proportions are the same, so the Variables, Gender and Juice are Independent**

**Ice Cream Study: Proportions are very different, so the Variables, Gender and Ice Cream are Associated (not Independent).**

	female		male			female		male	
	Count	Proportion	Count	Proportion		Count	Proportion	Count	Proportion
apple	28	0.35	14	0.35	chocolate	48	0.48	16	0.20
grape	12	0.15	6	0.15	strawberry	28	0.28	40	0.50
orange	40	0.50	20	0.50	vanilla	24	0.24	24	0.30
Total	80	1.00	40	1.00	Total	100	1.00	80	1.00

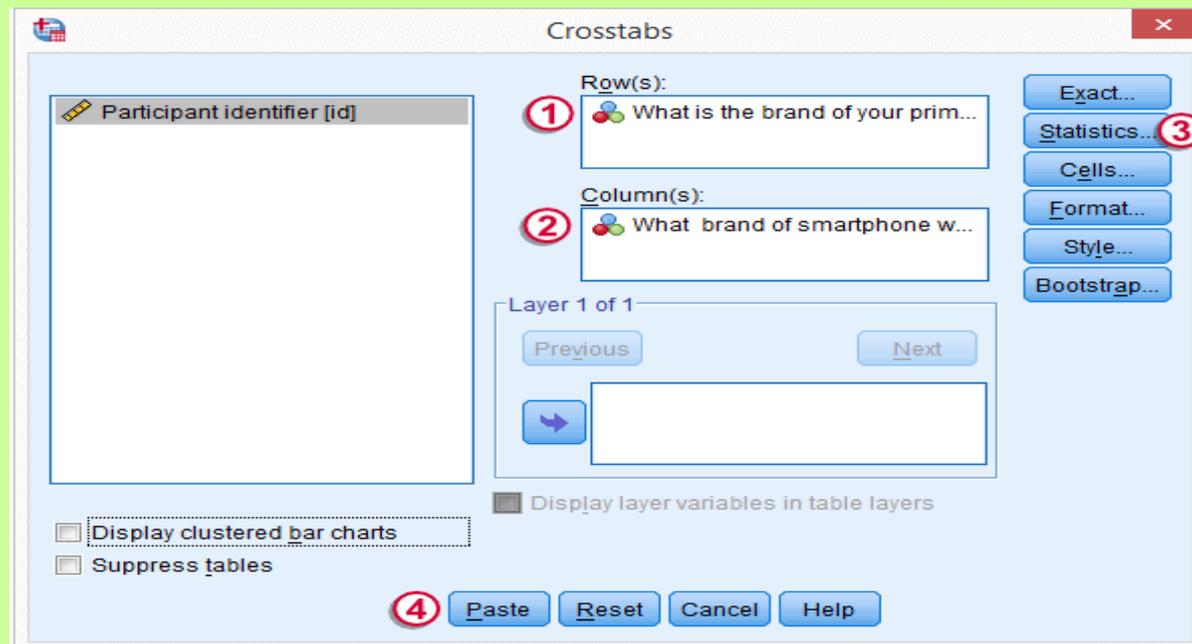
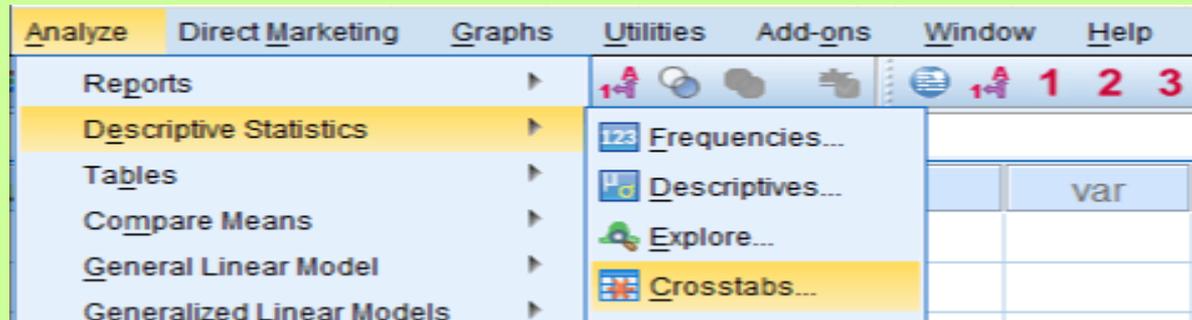
# Chi-Square

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

# Chi-Square

- **Null Hypothesis:** there is no Association, that is, the Variables are Independent

# Chi-Square



# Chi-Square

**What is the brand of your primary smartphone? ^ What brand of smartphone would you like have? Crosstabulation**

Count

		What brand of smartphone would you like have?				Total
		Samsung	HTC	Apple	Other	
What is the brand of your primary smartphone?	Samsung	27	11	4	8	50
	HTC	2	25	0	3	30
	Apple	3	2	17	8	30
	Other	8	2	4	26	40
Total		40	40	25	45	150

**Chi-Square Tests**

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	131.206 <sup>a</sup>	9	.000
Likelihood Ratio	115.888	9	.000
Linear-by-Linear Association	35.269	1	.000
N of Valid Cases	150		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 5.00.

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# Chi-Square

- **Fisher's exact test** is used when you have two nominal variables. A data set like this is often called an "R×C table," where R is the number of rows and C is the number of columns. Fisher's exact test is more accurate than the chi-squared test or G-test of independence when the expected numbers are small

# Chi-Square

- The null hypothesis is that the relative proportions of one variable are independent of the second variable

# Chi-Square

- **McNemar's test** is used when the two samples are not independent, but instead are two sets of observations on the same individuals (equivalent to the paired t-Test but for nominal variables)